Parcijalni izvodi

32. Naći parcijalne izvode 1. reda funkcija:

a) $f(x,y) = \ln(x+y^2)$

$$\frac{\partial f}{\partial x} = \frac{1}{x+y^2} \cdot 1 = \frac{1}{x+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x+y^2} \cdot 2y = \frac{2y}{x+y^2}$$

b) $f(x,y) = \dfrac{x^2+y^2}{x \cdot y^2}$

$$\frac{\partial f}{\partial x} = \frac{1}{y^2} \cdot \frac{2x^2-(x^2+y^2)\cdot 1}{x^2} = \frac{1}{y^2} \cdot \frac{x^2-y^2}{x^2} = \frac{1}{y^2} - \frac{1}{x^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x} \cdot \frac{2y^3-(x^2+y^2)\cdot 2y}{y^4} = -\frac{2x}{y^3}$$

c) $f(x,y,z) = \left(\dfrac{x}{y}\right)^z$

$$\frac{\partial f}{\partial x} = \frac{1}{y^z} \cdot z \cdot x^{z-1}$$

$$\frac{\partial f}{\partial y} = x^z \cdot (-z) \cdot y^{-z-1} = -\frac{z \cdot x^z}{y^{z+1}}$$

$$\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^z \cdot \ln\frac{x}{y}$$

33. Ako je $f(x,y) = \operatorname{arctg}\dfrac{y}{x}$ odrediti:

$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot y \cdot \left(-\frac{1}{x^2}\right) = -\frac{\frac{y}{\cancel{x^2}}}{\frac{x^2+y^2}{\cancel{x^2}}} = -\frac{y}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} \cdot 1 = \frac{\frac{1}{\cancel{x}}}{\frac{x^2+y^2}{x^{\cancel{2}}}} = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = -y \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot 2x =$$

$$= \frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = x \cdot (x^2+y^2)^{-2} \cdot (-1) \cdot 2y = -\frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)}$$

34

$f(x,y)$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$ → totalni diferencijal prvog reda

$$\Delta f = f(x_0+\Delta x, y_0+\Delta y) - f(x_0, y_0)$$

$$\Delta f \approx df(x_0, y_0)$$

$$df(x_0,y_0) = \frac{\partial f(x_0,y_0)}{\partial x} dx + \frac{\partial f(x_0,y_0)}{\partial y} dy$$

→ Naći diferencijal funkcije:

a) $f(x,y) = \frac{xy}{x-y}$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

$$\frac{\partial f}{\partial x} = \frac{y(x-y) - xy}{(x-y)^2} = y\frac{x-y-x}{(x-y)^2} = -\frac{y^2}{(x-y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x\cdot(x-y)+xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$df = -\frac{y^2}{(x-y)^2}\,dx + \frac{x^2}{(x-y)^2}\,dy$$

b) $f(x,y,z) = \dfrac{z}{x^2+y^2}$

$$\frac{\partial f}{\partial x} = z\cdot(-1)(x^2+y^2)^{-2}\cdot 2x = -\frac{2xz}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = z(-1)(x^2+y^2)^{-2}\cdot 2y = -\frac{2zy}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial z} = \frac{1}{x^2+y^2}\cdot z' = \frac{1}{x^2+y^2}$$

$$df = \frac{\partial f}{\partial x}\cdot dx + \frac{\partial f}{\partial y}\cdot dy + \frac{\partial f}{\partial z}\cdot dz$$

$$df = -\frac{2xz}{(x^2+y^2)^2}\,dx - \frac{2yz}{(x^2+y^2)^2}\cdot dy + \frac{1}{x^2+y^2}\,dz$$

Napomena: $d^2 f = d(df)$

diferencijal

izvod po promjenljivoj puta priraštaj

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\cdot dy$$

$$d^2 f = d\left(\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\cdot dy\right) =$$

$$= \left(\frac{\partial^2 f}{\partial x^2}\cdot dx + \frac{\partial^2 f}{\partial x\,\partial y}\cdot dy\right)\cdot dx + \left(\frac{\partial^2 f}{\partial y\,\partial x}\,dx + \frac{\partial^2 f}{\partial y^2}\,dy\right)\cdot dy =$$

$$= \frac{\partial^2 f}{\partial x^2}\cdot dx^2 + \frac{2\,\partial^2 f}{\partial x\,\partial y}\,dx\,dy + \frac{\partial^2 f}{\partial y^2}\cdot dy^2$$

35 Naći drugi diferencijal funkcije

$f(x,y) = x^3 + y^3 - 3x^2y + 3xy^2$

$d^2x = \frac{\partial^2 f}{\partial x^2} \cdot dx^2 + 2 \cdot \frac{\partial^2 f}{\partial x \partial y} \cdot dx\,dy + \frac{\partial^2 f}{\partial y^2} \cdot dy^2$

$\frac{\partial f}{\partial x} = 3x^2 - 6xy + 3y^2$

$\frac{\partial f}{\partial y} = 3y^2 - 3x^2 + 6xy$

$\frac{\partial^2 f}{\partial x^2} = 6x - 6y$

$\frac{\partial^2 f}{\partial y^2} = 6y + 6x$

$\frac{\partial^2 f}{\partial x \partial y} = -6x + 6y$

$d^2 f =$
$= 6(x-y)\,dx^2 + 12(y-x)\,dx\,dy +$
$+ 6(x+y)\,dy^2$

36 Približno izračunati: $1{,}002 \cdot (2{,}003)^2 \cdot (3{,}004)^3$

Neka je $f(x,y,z) = x \cdot y^2 \cdot z^3$

$x_0 = 1 \quad y_0 = 2 \quad z_0 = 3$

$\Delta x = 0{,}002 \quad \Delta y = 0{,}003 \quad \Delta z = 0{,}004$

$\rightarrow f(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z) \approx f(x_0, y_0, z_0) + df(x_0, y_0, z_0)$

$\Delta f \approx df$

$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy + \frac{\partial f}{\partial z} \cdot dz$

$f(x,y,z) = xy^2 \cdot z^3 \rightarrow f(1,2,3) = 108$

$\frac{\partial f}{\partial y} = 2xyz^3 \rightarrow 108$

$\frac{\partial f}{\partial x} = y^2 z^3 \rightarrow 108$

$\frac{\partial f}{\partial z} = 3xy^2z^2 \rightarrow 108$

$f(1{,}002;\ 2{,}003,\ 3{,}004) \approx$
$\approx f(1,2,3) + df(1,2,3) \approx$
$\approx 108 \cdot 1{,}009$

AGATHA RUIZ DE LA PRADA

(37) Naći $\frac{\partial f}{\partial x}$ i $\frac{\partial f}{\partial y}$ ako je $f(u,v) = U^2 \cdot \ln V$,

a $U = \frac{x}{y}$; $V = 3x - 2y$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial U} \cdot \frac{\partial U}{\partial x} + \frac{\partial f}{\partial V} \cdot \frac{\partial V}{\partial x}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial U} \cdot \frac{\partial U}{\partial y} + \frac{\partial f}{\partial V} \cdot \frac{\partial V}{\partial y}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial U} \cdot \frac{1}{y} + \frac{\partial f}{\partial V} \cdot 3 = 2U \ln V \cdot \frac{1}{y} + U^2 \cdot \frac{1}{V} \cdot 3 =$$

$$= \ln(3x-2y) \frac{2x}{y^2} + 3 \frac{x^2}{y^2} \cdot \frac{1}{\ln(3x-2y)}$$

$$\frac{\partial f}{\partial y} = -2U \cdot \ln V \cdot x \cdot \frac{1}{y^2} + U^2 \cdot \frac{1}{V} \cdot (-2) =$$

$$= -2 \frac{x^2}{y^3} \cdot \ln(3x-2y) - 2 \frac{x^2}{y^2} \cdot \frac{1}{\ln(3x-2y)}$$

(38) Naći $d^2 f$ za $f = f(U, V)$

$U = x + y$

$V = x - y$

$$d^2 f = \frac{\partial^2 f}{\partial x^2} dx^2 + 2 \frac{\partial^2 f}{\partial x \partial y} dx\, dy + \frac{\partial^2 f}{\partial y^2} \cdot d^2 y$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial U} \cdot \frac{\partial U}{\partial x} + \frac{\partial f}{\partial V} \cdot \frac{\partial V}{\partial x}$$

nemamo kako f od U, V izgleda

$$\frac{\partial u}{\partial x} = 1 \; ; \quad \frac{\partial v}{\partial x} = 1 \longrightarrow \boxed{\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = 1 \; ; \quad \frac{\partial v}{\partial y} = -1 \longrightarrow \boxed{\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}\right)$$

$$\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} = g(u,v)$$

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial g}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\to \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial u} \cdot \underbrace{\left(\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}\right)}_{g} \cdot \underbrace{\frac{\partial u}{\partial x}}_{1} + \frac{\partial}{\partial v} \cdot \left(\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}\right) \cdot \frac{\partial v}{\partial x}$$

$$= \left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v \partial u}\right) \cdot 1 + \left(\frac{\partial^2 f}{\partial v \cdot \partial u} + \frac{\partial^2 f}{\partial v^2}\right) \cdot 1 =$$

$$= \frac{\partial^2 f}{\partial u^2} + 2\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2} \quad \boxed{\frac{\partial^2 f}{\partial x^2}}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}\right)$$

neka je $h(u,v) = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial v}$

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial h}{\partial v} + \frac{\partial v}{\partial x}$$

AGATHA RUIZ DE LA PRADA

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial U} \underbrace{\left(\frac{\partial f}{\partial U} - \frac{\partial f}{\partial V}\right)}_{h} \cdot \underbrace{\frac{\partial U}{\partial x}}_{1} + \frac{\partial}{\partial V} \cdot \left(\frac{\partial f}{\partial U} - \frac{\partial f}{\partial V}\right) \cdot \underbrace{\frac{\partial V}{\partial x}}_{1} =$$

$$= \frac{\partial^2 f}{\partial U^2} - \cancel{\frac{\partial^2 f}{\partial U \partial V}} + \cancel{\frac{\partial^2 f}{\partial U \partial V}} - \frac{\partial^2 f}{\partial V^2} = \frac{\partial^2 f}{\partial U^2} - \frac{\partial^2 f}{\partial V^2}$$

neka je: $\frac{\partial^2 f}{\partial U^2} = f''_{uu}$; $\frac{\partial^2 f}{\partial U \partial V} = f''_{UV}$; $\frac{\partial^2 f}{\partial V^2} = f''_{vv}$

$$d^2 f = (f''_{UU} + 2f''_{UV} + f''_{vv})\, dx^2 + 2(f''_{UU} - f''_{vv}) \cdot dx\,dy + \frac{\partial^2 f}{\partial y^2} \cdot dy^2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \underbrace{\left(\frac{\partial f}{\partial U} - \frac{\partial f}{\partial V}\right)}_{m(U,V)} = \frac{\partial m}{\partial U} \cdot \frac{\partial U}{\partial y} + \frac{\partial m}{\partial V} \cdot \frac{\partial V}{\partial y} =$$

$$= \frac{\partial}{\partial U}\left(\frac{\partial f}{\partial U} - \frac{\partial f}{\partial V}\right) \cdot \underbrace{\frac{\partial U}{\partial y}}_{1} + \frac{\partial}{\partial V}\left(\frac{\partial f}{\partial U} - \frac{\partial f}{\partial V}\right) \cdot \underbrace{\frac{\partial V}{\partial y}}_{-1} =$$

$$= \frac{\partial^2 f}{\partial U^2} - \frac{\partial^2 f}{\partial U \partial V} - \frac{\partial^2 f}{\partial U \partial V} + \frac{\partial^2 f}{\partial V^2} = \frac{\partial^2 f}{\partial U^2} - 2\frac{\partial^2 f}{\partial U \partial V} + \frac{\partial^2 f}{\partial V} =$$

$$= f''_{UU} - 2f''_{UV} + f''_{vv}$$ → množi se sa dy^2 kad se uvrsta u diferencijal

$$d^2 f = (dx^2 + 2dx\,dy + dy^2) f''_{uu} + 2(dx^2 - dy^2) f''_{UV} + (d^2x - 2dx\,dy + dy^2) f''_{vv} =$$

$$= (dx + dy)^2 f''_{UU} + 2(dx^2 - dy^2) f''_{UV} + (dx - dy)^{2} f''_{vv}$$

(39) $f = f(U, V)$; $U = x^3 + y^2$; $V = 4x^2 - 2y^3$

$$d^2 f = \frac{\partial^2 f}{\partial x^2} \cdot dx^2 + 2 \frac{\partial^2 f}{\partial x \partial y} dx\, dy + \frac{\partial^2 f}{\partial y^2} \cdot dy^2$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial U} \cdot \frac{\partial U}{\partial x} + \frac{\partial f}{\partial V} \cdot \frac{\partial V}{\partial x} = \frac{\partial f}{\partial U} \cdot 3x^2 + \frac{\partial f}{\partial V} \cdot 8x$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \underbrace{\left(\frac{\partial f}{\partial U} \cdot 3x^2 + \frac{\partial f}{\partial V} \cdot 8x \right)}_{g} =$$

→ izvod proizvoda

parc. izvod po x

$$= 6x \frac{\partial f}{\partial U} + 3x^2 \cdot \left(\frac{\partial^2 f}{\partial U^2} \cdot \frac{\partial U}{\partial x} + \frac{\partial^2 f}{\partial V \cdot \partial U} \cdot \frac{\partial V}{\partial x} \right) +$$

$$\boxed{+ 8 \cdot \frac{\partial f}{\partial V} + 8x \cdot \left(\frac{\partial^2 f}{\partial U \cdot \partial V} \cdot \frac{\partial U}{\partial x} + \frac{\partial^2 f}{\partial V^2} \cdot \frac{\partial V}{\partial x} \right)} =$$

$$= 6x \frac{\partial f}{\partial U} + 3x^2 \cdot \left(\frac{\partial^2 f}{\partial U^2} \cdot 3x^2 + \frac{\partial^2 f}{\partial V \cdot \partial U} \cdot 8x \right) +$$

$$+ 8 \cdot \frac{\partial f}{\partial V} + 8x \cdot \left(\frac{\partial^2 f}{\partial U \cdot \partial V} \cdot 3x^2 + \frac{\partial^2 f}{\partial V^2} \cdot 8x \right) =$$

$$= 6x\, f'_U + 9x^4 \cdot f''_{UU} + 24x^3 f''_{UV} + 8 f'_V +$$

$$+ 24x^3 \cdot f''_{UV} + 64x^2 \cdot f''_{VV} =$$

$$= 6x \cdot f'_U + 8 f'_V + 9x^4 f''_{UU} + 48x^3 f''_{UV} + 64x^2 f''_{VV}$$

40. Pod pretpostavkom da su g i h diferencijabilne dovoljan broj puta, izračunati:

$$x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \cdot \frac{\partial^2 u}{\partial x \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} = 0, \text{ ako}$$

je $u = g\left(\frac{y}{x}\right) + x \cdot h\left(\frac{y}{x}\right)$

Neka je $\frac{y}{x} = t$. Tada funkcije g i h posmatramo kao f-je $g = g(t)$ i $h = h(t)$. Uvedimo oznake

$$g' = g'_t \; ; \; g'' = g''_t \quad i \quad h' = h'_t \; ; \; h'' = h''_t$$

$$\frac{\partial u}{\partial x} = \frac{\partial g}{\partial t} \cdot \frac{\partial t}{\partial x} + h + x \cdot \frac{\partial h}{\partial t} \cdot \frac{\partial t}{\partial x}$$

$$\frac{\partial u}{\partial x} = g' \cdot \left(-\frac{y}{x^2}\right) + h + x \cdot h' \cdot \left(-\frac{y}{x^2}\right)$$

$$\frac{\partial u}{\partial x} = g' \frac{\partial t}{\partial y} + h + x \cdot h' \cdot \frac{\partial t}{\partial y} = \boxed{-\frac{y}{x^2} g' + h - \frac{y}{x} \cdot h'}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{y}{x^2} g' + h - \frac{y}{x} \cdot h'\right) =$$

$$= \frac{2y}{x^3} \cdot g' - \frac{y}{x^2} \cdot g'' \cdot \frac{\partial t}{\partial x} + h' \cdot \frac{\partial t}{\partial x} + \frac{y}{x^2} \cdot h' - \frac{y}{x} \cdot h'' \cdot \frac{\partial t}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2y}{x^3} g' + \frac{y^2}{x^4} \cdot g'' - \cancel{\frac{y}{x^2} h'} + \cancel{\frac{y}{x^2} h'} + \frac{y^2}{x^3} \cdot h''$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = \frac{2y}{x^3} \cdot g' + \frac{y^2}{x^4} \cdot g'' + \frac{y^2}{x^3} \cdot h''}$$

$$\frac{\partial U}{\partial y} = \frac{\partial g}{\partial t}\cdot\frac{\partial t}{\partial y} + x\cdot\frac{\partial h}{\partial t}\cdot\frac{\partial t}{\partial y} =$$

$\frac{1}{x}\cdot g' + h'$

$$= + g'\,\frac{y}{x^2} + x\cdot h'\cdot\frac{1}{x} = \frac{1}{x}\cdot g' + h'$$

$$\frac{\partial^2 U}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{1}{x}\cdot g' + h'\right) =$$

$$= \frac{1}{x}\cdot g''\cdot\frac{\partial t}{\partial y} + h''\cdot\frac{\partial t}{\partial y} = \frac{1}{x^2}\cdot g'' + \frac{1}{x}\cdot h''$$

$$\frac{\partial^2 U}{\partial x\,\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial y}\left(-\frac{y}{x^2}\cdot g' + h - \frac{y}{x}\cdot h'\right) =$$

$$= \frac{\partial}{\partial y}\left(-\frac{y}{x^2}\cdot g'\right) + \frac{\partial h}{\partial t}\cdot\frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(-\frac{y}{x}\cdot h'\right) =$$

$$= -\frac{g'}{x^2} - g''\cdot\frac{\partial t}{\partial y}\cdot\frac{y}{x^2} + h'\cdot\frac{\partial t}{\partial y} - \frac{h'}{x} + \cancel{\ldots} - \frac{y}{x}\cdot h''\,\frac{\partial t}{\partial y}$$

$\frac{1}{x}$ $\frac{1}{x}$ $\frac{1}{x}$

$$= -\frac{1}{x^2}\cdot g' - \frac{y}{x^3}\cdot g'' + \cancel{\frac{1}{x}\cdot h'} - \cancel{\frac{1}{x}\cdot h'} - \frac{y}{x^2}\cdot h'' =$$

$$\Rightarrow \frac{\partial^2 U}{\partial x\,\partial y} = -\frac{1}{x^2}\cdot g' - \frac{y}{x^3}\cdot g'' - \frac{y}{x^2}\cdot h''$$

→ Uvrstimo dobivene izvode u datu j-nu

$$\left(\cancel{\frac{2y}{x}\cdot g'} + \cancel{\frac{y^2}{x^2}\cdot g''} + \frac{y^2}{x}\cdot h''\right) - \cancel{2\frac{y}{x}\cdot g'} - \cancel{2\frac{y^2}{x^2}\cdot g''} -$$

$$\frac{2y^2}{x}\cdot h'' + \cancel{\frac{y^2}{x^2}\cdot g''} + \frac{y^2}{x}\cdot h'' = 0$$